A THERMODYNAMIC RELATION FOR ELASTIC SOLIDS

JOHN V. HARTER

A THERMODYNAMIC RELATION

FOR ELASTIC SOLIDS

* * * * *

John V. Harter

A THERMODYNAMIC RELATION
FOR ELASTIC SOLIDS

by

John V. Harter

Lieutenant, United States Navy

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1 9 6 2

A THERMODYNAMIC RELATION

FOR ELASTIC SOLIDS

by

John V. Harter

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

from the

United States Naval Postgraduate School

ABSTRACT

A postulated thermodynamic relation for elastic solids which equates the stress dependence of thermal expansion to the temperature dependence of reciprocal Young's modulus has been tentatively verified in earlier work by Rosenfield and Averbach (J. Ap. Phys., 27, 2 (1956)). This thesis describes experiments which were intended to extend this verification, but which succeeded only in verifying quite closely the results obtained earlier for the temperature dependence of the reciprocal Young's modulus. Experimental techniques utilizing a high temperature Tuckerman optical strain gage instead of the wire resistance strain gages employed by the earlier authors are described, and the difficulties encountered in their use are discussed.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction

If we presume that strain e depends only upon applied stress σ and upon the temperature T, and upon no other parameters such as time or stress history, i.e., $e = e(\sigma, T)$, a state of affairs which may be thought of as characterizing an elastic solid, and if we presume that this dependence is "sufficiently continuous", then

$$\frac{\partial}{\partial \sigma}\left[\partial e/\partial T\right] = \frac{\partial}{\partial T}\left[\partial e/\partial \sigma\right] .$$

The coefficient of linear expansion α is defined as $\partial e/\partial T$ for constant σ (σ is usually presumed to be zero), and, if the elasticity is linear, the rate of change $\partial e/\partial \sigma$ at constant temperature is the reciprocal of the Young's modulus. Thus, the preceding equation becomes $\partial \alpha/\partial \sigma\big]_T = d(1/E)/dT$ where the ordinary, rather than partial, derivative has been written on the right, since E is not a function of σ. However, α is a function of both T and σ (i.e., $\alpha = \alpha(T, \sigma)$) and the variation with respect to σ, a little recognized property, results from the fact that for most materials E is a function of T.

In the preceding discussion and in the experimental work which is reported later in this thesis, the state of stress is taken to be uniaxial. However, it would not be difficult to modify the preceding equations so as to describe a general state of stress.

In 1956, Rosenfield and Averbach verified the above relationship within their limits of experimental accuracy and utilized the theory that α would vary with stress to determine a "true elastic limit"[1] *

*Numbers in boxes refer to the Bibliography which appears on Page 3_.

for 1020 steel. These authors also stated that the expansion coefficient of steel under tensile load increases linearly with stress until the elastic limit is reached. N. F. Kunin and V. N. Kunin [2] also studied this change in expansion coefficient with stress for cold-worked copper and found the coefficient to be linear with load.

In both cases a very small temperature range was used (Rosenfield and Averbach 13°C to 28°C; Kunin and Kunin 14.98°C to 41.48°C).

It was the purpose of the present study to attempt to expand the range of experimentation to higher temperatures and compare the results to those of Rosenfield and Averbach.

2. Description of Apparatus

A. Stress Application and Measurement

Stress was applied to the specimen by means of a dead load testing machine, as shown in Figures 1 and 2. The capability of holding a constant load and the proximity of installed furnace temperature control units made it more desirable for this application than a standard hydraulically operated tensile testing machine. The machine was loaded by means of dead weights suspended from the end of the lever arm system. The lever arm rested on two case hardened knife edges. The upper swivel block also rested on a case hardened knife edge. A universal joint was connected to the upper swivel block, the lower end of which held an SR-4 type U load cell. A turnbuckle was connected to the lower end of the load cell. The upper universal joint and the double universal joint on the lower end of the system insured that the system had adequate freedom to prevent imposition of bending, torsional, or shear loadings. Following the upper turnbuckle was the upper pulling grip, the specimen, the lower pulling grip, a turnbuckle and the lower universal joint. This system was connected at the bottom by bolting the lower end of the universal joint to the base of the dead load machine. A movable lead counterweight which balanced the lever system, was located on a screw extending from the front of the lever arm.

Measurement of the load was accomplished with a Baldwin SR-4 Type U load cell of 2,000 pounds capacity in conjunction with an SR-4 Type

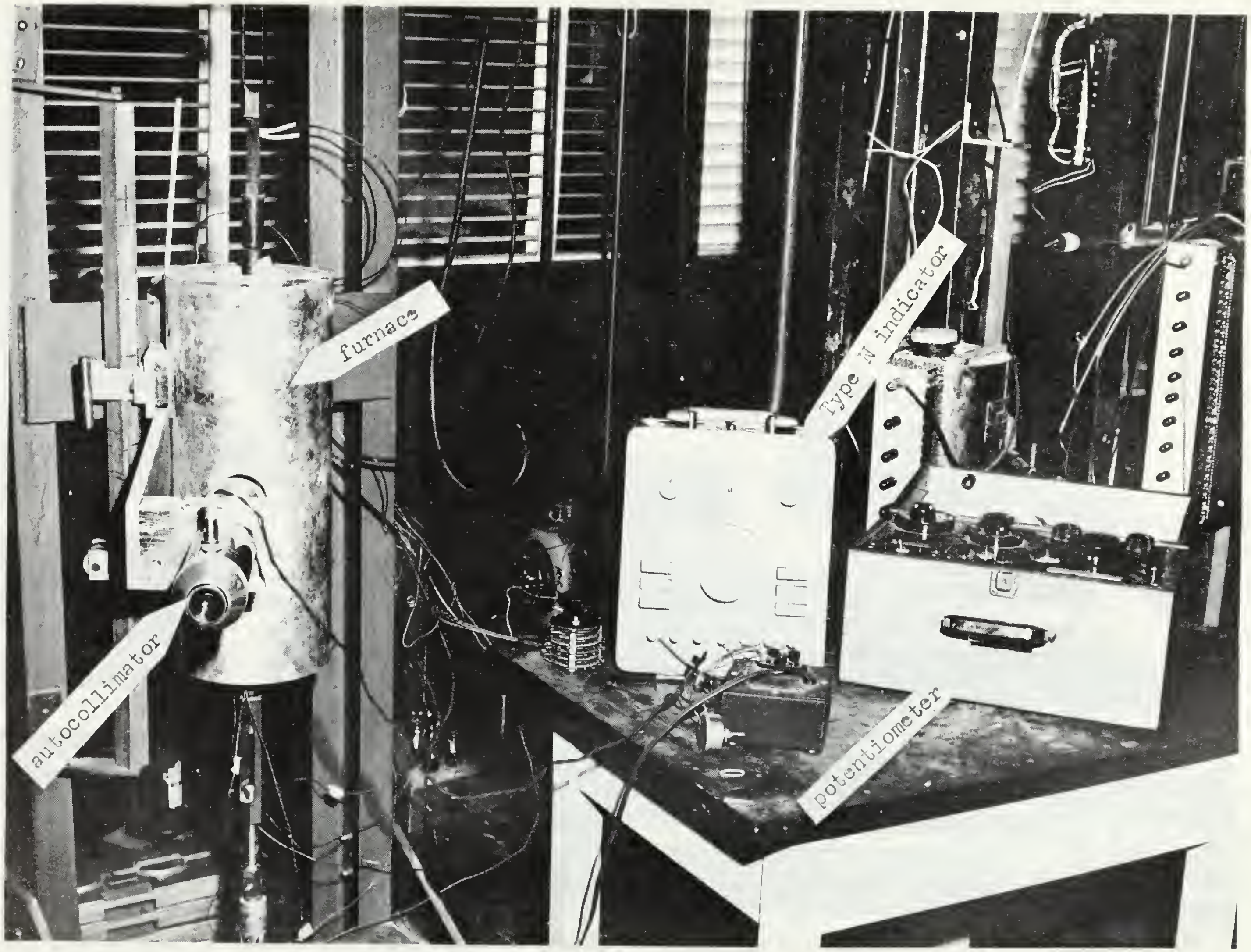

Figure 1. Photograph of General Arrangement of Equipment with Furnace in place for Testing.

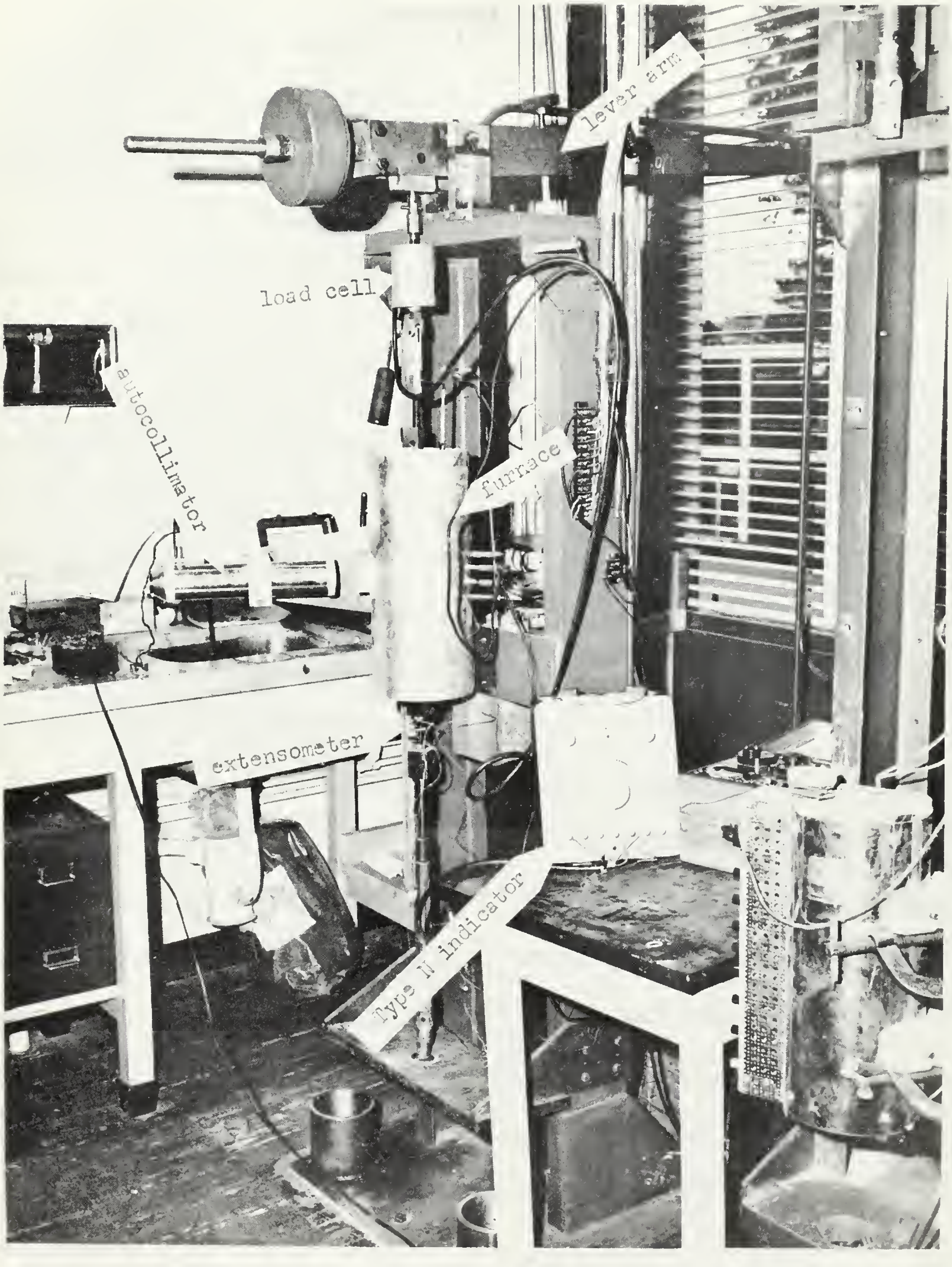

Figure 2. Photograph of General Arrangement of Equipment with Furnace Raised to show Specimen.

a strain indicator. The load cell was mounted as a part of the linkage, as described above.

B. Strain Measurement

A Tuckerman autocollimator No. B-366 and a Tuckerman optical extensometer, both manufactured by the American Instrument Company, Silver Spring, Maryland, were used to measure the strain.

The Tuckerman autocollimator consists of an objective lens system, a reticle consisting of a fiducial spot and a scale, a light source, and an eyepiece. A simplified diagram of the optical system is shown in Figure 4. Light from the fiducial mark is directed to the objective lens from which it emerges as parallel light. The beam is reflected from the extensometer mirror system back through the objective lens which focuses the light on the reticle. The reticle lies in the focal plane of the objective lens, which is also the plane of the real image. The magnified reticle scale and the image of the fiducial mark are therefore visible through the eyepiece.

The extensometer is one that has been developed by the American Instrument Company and the National Bureau of Standards for use at temperatures up to 500°F. The body of the gage is made of cold-rolled steel and the lozenge is made of Star "J" metal, manufactured by the Haynes Stellite Co. An 0.4 inch lozenge was used and rotates in a 110° notch. The gage was held in place on the specimen by a light spring. (See Figure 3).

Readings taken with the autocollimator perpendicular to the gage

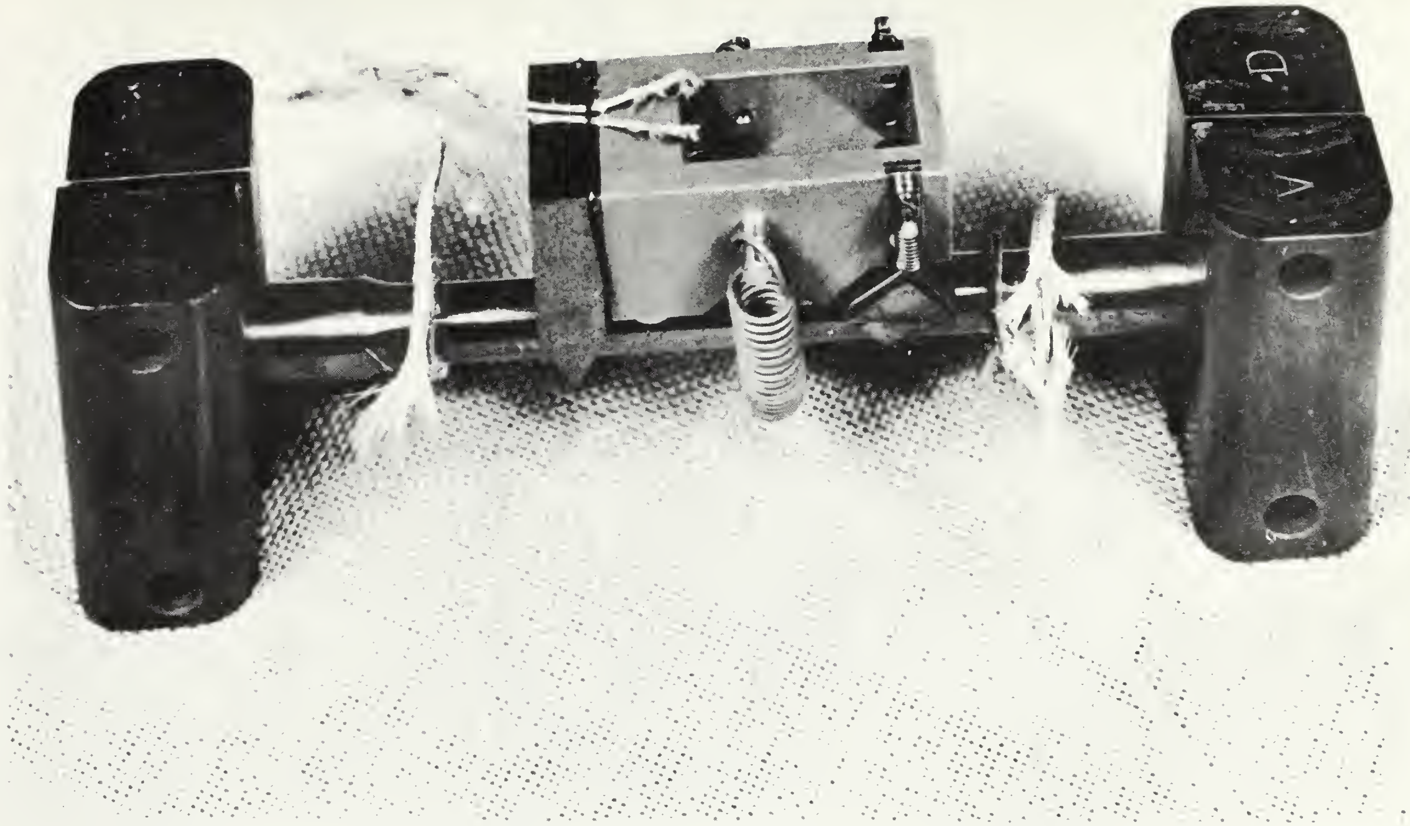

Figure 3. Photograph of Extensometer Mounted on Specimen.

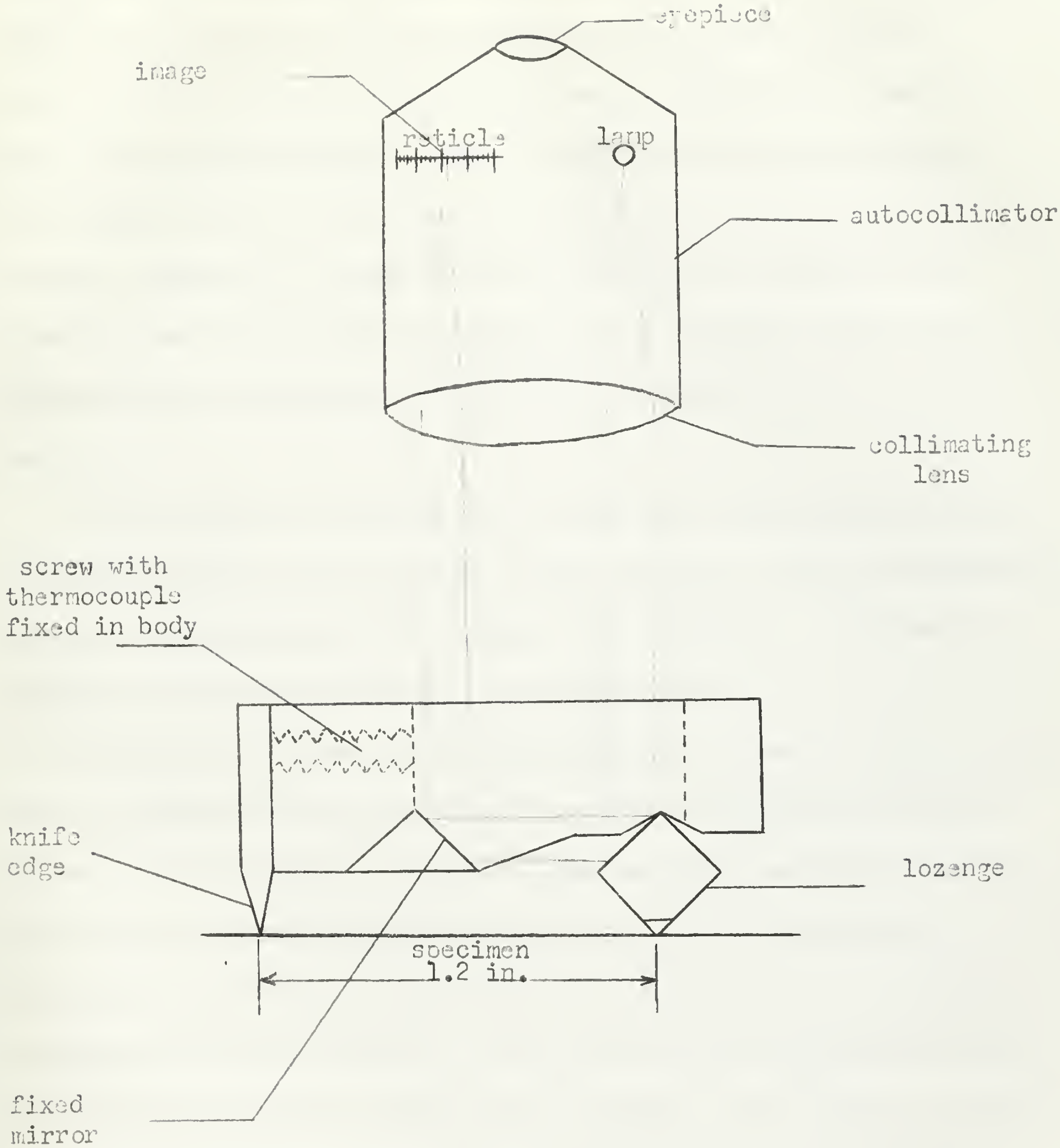

Figure 4. Fundamental optical diagram of the Tuckerman optical strain gage.

depend only on the angle between the two mirrors. If the autocollimated axis deviates from the perpendicular plane, the light will move at right angles to the scale and make it difficult to read the gage. The standard optical strain gage for use at room temperature has a glass roof prism in place of a fixed mirror and reduces this effect. However, the glass roof prism cannot be utilized at the higher temperatures because it would fail by cracking, [3] and the autocollimator must therefore be fixed in position in order to read the gage.

A fixed clamp for holding the autocollimator was manufactured and fixed to the body of the furnace so that the axis of the autocollimator remained perpendicular to the furnace. (See Figure 3). Alignment of the gage and autocollimator was thereby simplified.

The decision to use the Tuckerman optical system for strain measurement was made after consideration of various unconventional devices and resistance wire strain gages. Since the work previously done in this field was accomplished with resistance wire strain gages, a discussion of this type of measurement is in order. H. Muir [4] used resistance wire strain gages in showing the effect of residual strain on the thermal expansion coefficient of steel. In his work, Dr. Muir covered a temperature range of 60.8°F to 111.2°F. A. R. Rosenfield and B. L. Averbach in their work covered a range of only 55.4°F to 82.4°F. The experimental method used in both examinations was developed by Dr. Muir and requires the use of invar for mounting the dummy

gage. With invar being used, it is easy to see that the strain in the specimen due to thermal stress is measureable and that temperature compensation for changes in the gage alone is effected. This system is feasible for the temperature range covered; however, at temperatures above 120°F invar loses its invariant property and would no longer be effective in giving the required type of temperature compensation.

The author studied the possibility of using fused quartz tubing as the dummy specimen and making allowance for its expansion arithmetically. It is believed that this method is the most feasible if resistance wire strain gages were to be used. Other characteristics of the resistance wire strain gage enter into the decision, however. The gage factor would change considerably over a temperature range of 300 to 400°, requiring another correction, and reliable data on the thermal characteristics of the gage would have to be available. Finally, the difficulties involved in mounting the wire strain gages accurately conclude the list of disadvantages for this application. (H. Muir was able to obtain excellent reproducibility of expansion data with precision being of the order of $\pm$ 0.1 x 10^{-6} in./in.-°C).

Although many special cases of high temperature uses of resistance wire strain gages were found in the literature [5, 6, 7, 8], in addition to the factors previously listed, it was felt that these methods were unsuitable due to lack of commercially available equipment (the gages being of laboratory construction) and insufficient time and equipment to attempt their manufacture in our own laboratories.

C. Calibration

The extensometer was calibrated to find the gage factor at room temperature by comparison with a standard Tuckerman extensometer for which the gage factor was known. A tensile test of a specimen was conducted with the standard extensometer mounted on it and subsequently with the high temperature extensometer. A multiplication factor was applied to the value of E found with the high temperature extensometer to equate it to the value of E found with the standard extensometer. This multiplier was used throughout as the gage factor. The change in this gage factor at higher temperatures is stated by the manufacturer as less than one per cent at 500°F. This was determined by P. R. Weaver of the National Bureau of Standards by use of a transfer system accurate to one per cent, in which the high temperature gage was compared with the standard gage for temperatures up to 500°F.

In order to cycle the temperature of the specimen under constant load and obtain the coefficient of expansion of the specimen, an additional calibration curve was necessary to account for expansion of the gage body. This calibration was found to be necessary after attempting to determine the coefficient of expansion by use of uncorrected extensometer readings. The slopes of the various expansion curves at constant load were not indicative of known values of the expansion coefficient for the specimen material. However, it was believed that this would not alter the results since the change

in the slope with stress was the desired quantity and this difference should be independent of the error due to the expansion of the gage body. As long as the temperature of the gage body maintained the same rate of temperature change upon cooling as the specimen, this would be true.

A thermocouple was mounted in the gage body. A test run indicated that the gage body temperature in fact did vary in its relation to the specimen temperature due to its closer proximity to the viewing port; therefore some type of correction was deemed necessary. It was then determined that if the temperature of the gage body at each reading was multiplied by an appropriate calibration factor, the true expansion curve could be determined.

Let $A = B + C\Delta T$

where A is the corrected reading

B is the actual extensometer reading indicating the relative expansion between the specimen and gage body

C is a calibration factor and ΔT is the change in gage body temperature from the first reading of the test run.

The appropriate calibration factor therefore would have to be such that the product $C\Delta T$ resulted in the true expansion of the gage body.

To determine the calibration curve it was decided to conduct an expansion test on a material of known thermal coefficient of expansion and, ideally, one having a very low coefficient so that any errors in

the given data of the material would be small and the largest percentage of the extensometer readings would be due to the expansion of the gage body. Fused quartz would fill this need, but was not readily available, so Vycor, which was on hand, was substituted.

Vycor is a brand of glass manufactured by the Corning Glass Company. [9] It is composed of approximately 96% silica, the remainder being mostly boric oxide, and it may be compared with fused quartz in properties and performance. The Vycor brand glass starts out as a glass of normal characteristics and is then processed to remove practically all constituents except silica. It is then fired at high temperature to complete the process. The linear coefficient of thermal expansion is stated by the manufacturer to be 8×10^{-7} in./in.-^{o}C.

Two thermocouples were attached to the surface of a piece of Vycor with Sauereisen cement. The extensometer was then attached and the assembly was positioned in the furnace. Extensometer reading, extensometer temperature, and specimen temperature were recorded throughout a temperature range of 450^{o}F down to room temperature to conduct the calibration test. The calibration curve was produced by multiplying the extensometer readings by the constant that would give strain readings at a constant temperature (3.343×10^{-4} in./in./division) and plotting these on graph paper. The expansion curve for Vycor was plotted on the same paper and added to the

extensometer strain readings. The slope of the resulting curve was determined to be 9.92×10^{-6} in./in.-°F and was utilized as the calibration factor.

D. Temperature Control and Measurement

The furnace was an air type furnace having 27 feet of No. 17 Nichrome wire wound around a refractory tube 2.5 inches inside diameter. A 2-inch diameter viewing port was placed in the side of the furnace and sealed with a pyrex window. The overall diameter of the furnace, including the insulation and exterior shell, measured 7 inches.

The ends of the tube were sealed around the linkage of the dead load machine by use of layers of glass wool insulation. A Leeds and Northrup temperature recording controller, in conjunction with a duration adjusting temperature control unit, held individual thermocouple readings to a variation of $\pm$ 2°F with time, although there were spatial gradients of about $\pm$ 5° maximum even after steady state had been achieved. The effect of the spatial variations was minimized by using average values; see Page 16.

Two Chromel-Alumel thermocouples were placed on the specimen as described in the section entitled "Experimental Procedure", Page 15. In addition, a Chromel-Alumel thermocouple was silver soldered in a threaded piece of brass and screwed into the body of the extensometer. The output of these thermocouples was read by a potentiometer, Model 2732, manufactured by Rubicon Instruments, Philadelphia, Pennsylvania.

3. Procedure

Experimental Procedure

The diameter of the specimen was measured at three locations on the gage length by means of a micrometer to the nearest 0.0001 inches. The average area computed from these readings was utilized in determining the stress on the specimen during tests.

Two holes, one at each end of the specimen gage length, but outside the extensometer gage length, were drilled in the surface of the specimen. A Chromel-Alumel thermocouple was placed in each hole and the edges of the hole were peened over to hold the thermocouple in place.

The specimen was then placed in the grips of the dead-load machine, and the extensometer was attached by use of a spring mounted on the extensometer and encircling the specimen. The furnace was then lowered into place and the autocollimator was sighted on the extensometer through the viewing port and mounted in place.

Tensile tests were conducted while the specimen was held at temperatures of approximately 80, 150, 250, 350 and 450°F. At least two tests were conducted at each temperature to determine reproducibility. Each run required approximately one and a quarter hours to complete after the specimen reached the testing temperature and contained at least 20 points per run.

The data recorded included the extensometer reading, the SR-4 Type N indicator reading of the load cell output, and the potentiometer

reading of the two thermocouples mounted on the specimen. The load cell versus extensometer readings obtained were subjected to a least squares analysis to determine the slopes of the lines (see Appendix I). A scaling factor was then applied to the value of the slope to determine E.

Thermal expansion measurements were conducted while the specimen was subjected to constant loads at approximately 5,000, 10,000, 15,000, 20,000 and 25,000 psi. The elongation was measured by observing the Tuckerman optical strain gage, while load and temperature readings were obtained as in the tensile tests. In addition, a thermocouple was mounted in the body of the extensometer and its output was also recorded. By knowing the temperature of the gage body, it was possible to apply a correction factor for the error caused by the expansion of the gage relative to the specimen. This correction factor was previously determined and obtained as described in Part C of the preceding chapter. The specimen and extensometer were heated to 450°F and then cooled in increments, allowing temperature equilibrium to be established at each point where temperature and extension readings were taken. This procedure was repeated for each load. Each run required a 24-hour period to complete in order to insure steady state at each level of an average of 12 temperature levels. The average temperature of the two thermocouples was utilized as the specimen temperature. The extensometer reading was corrected by means of the correction factor noted above. The corrected extensometer readings versus temperature readings were then subjected to a least

squares analysis to determine the slopes of the lines.

4. Results

Table I lists the results obtained in this work and by Rosenfield and Averbach.

TABLE I

	Temperature Range	$d(1/E)/dT$	$\partial\alpha/\partial\sigma]_T$
		(units of 10^{-11} in.^{2}/lb-°C)	
Rosenfield and Averbach	55.4-82.4°F	1.2	0.6
(two specimens)	55.4-82.4°F	0.8	1.0
This Study	78.0-450°F	$1.02 \pm .12$	4.53 ± 1.57

Graphical results are shown on the following two pages. The plots shown utilize the symbol ⟟, which indicates the point obtained from a statistical study (see Appendix I) with one standard deviation above and below the most probable value being indicated by the length of the line through the dot.

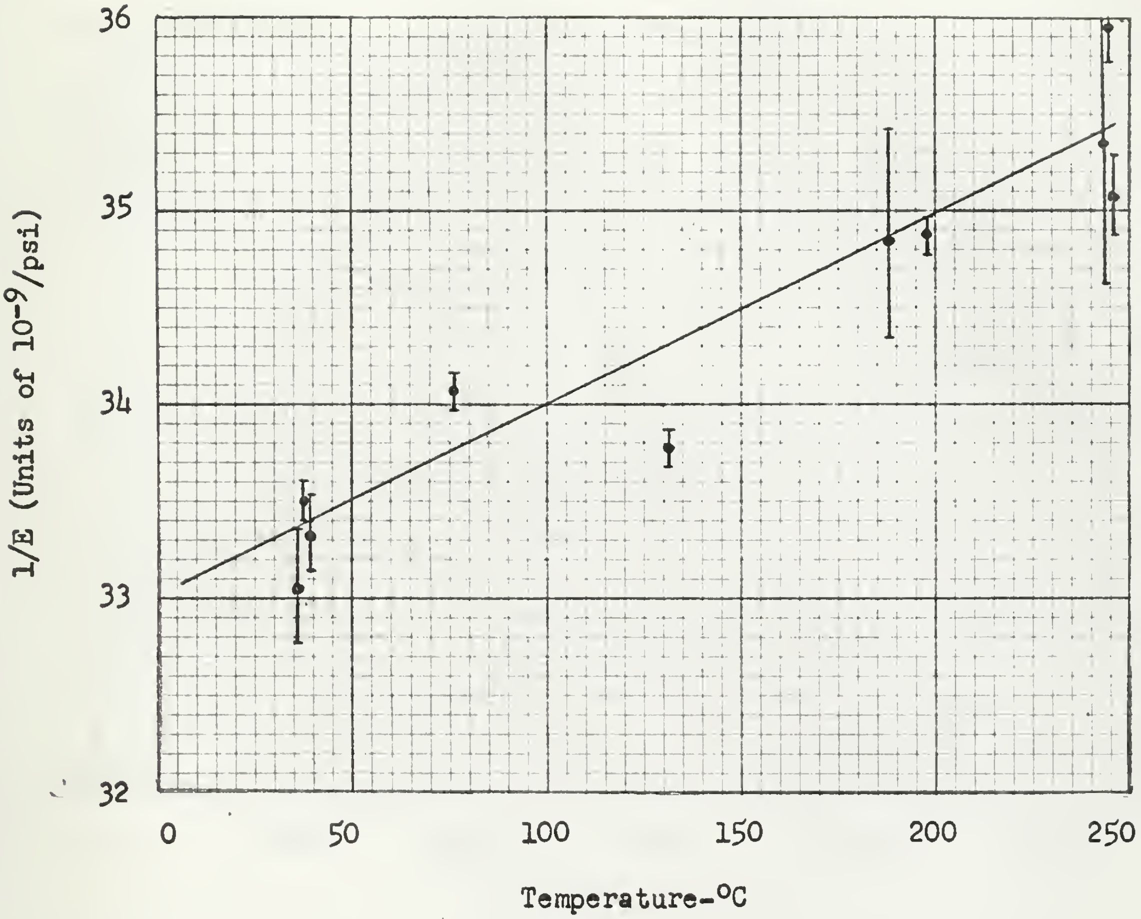

Figure 5. 1/E versus temperature. Values shown were determined from a statistical study of experimental points of stress-strain tests at constant temperatures (see Appendix I). Dot in center of ⫿ indicates most probable value. One standard deviation above and below most probable value is indicated by vertical line through dot.

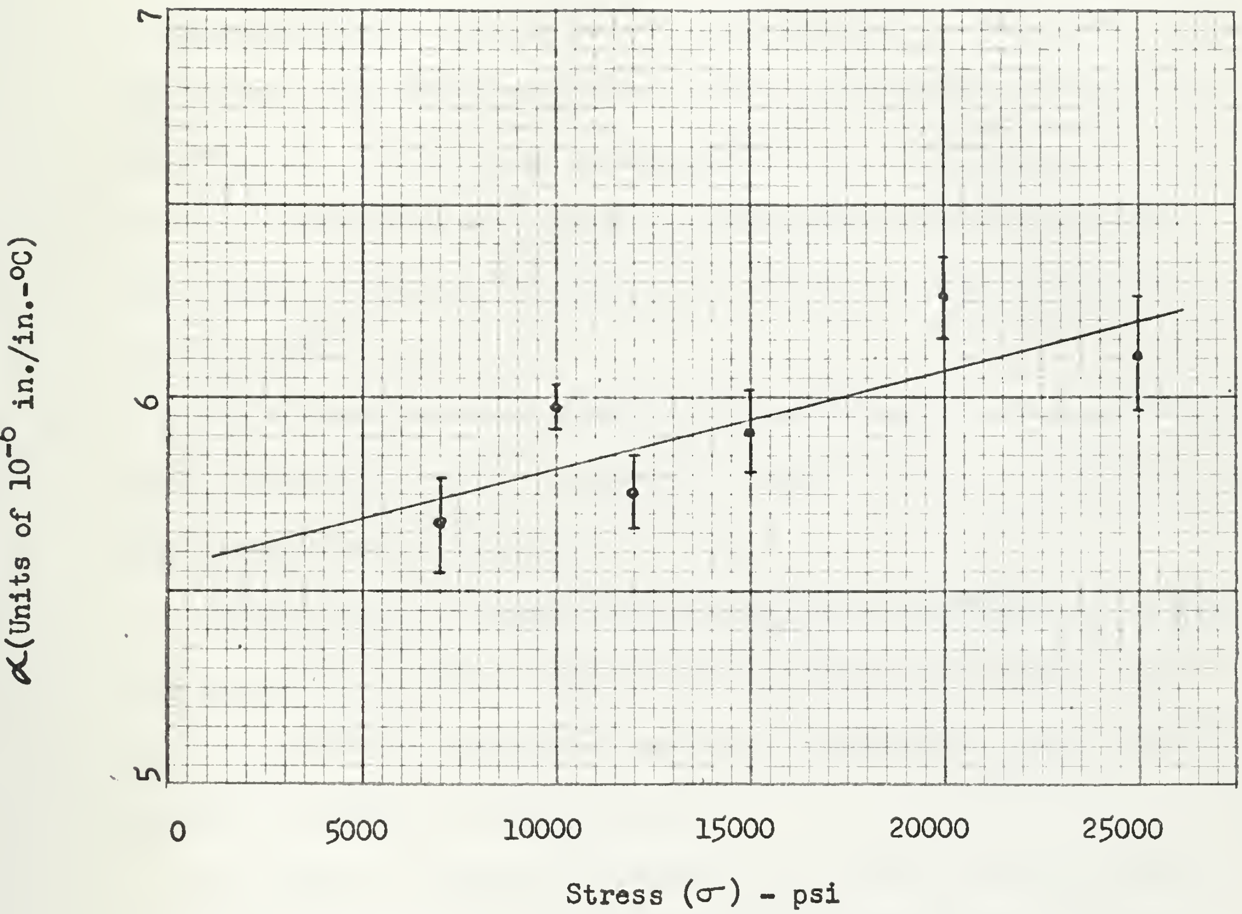

Figure 6. Coefficient of expansion (α) versus stress (σ). Values shown were determined from a statistical study of experimental points of strain versus temperature curves at constant stress (see Appendix I). Dot in center of I indicates most probable value. One standard deviation above and below most probable value is indicated by vertical line through dot.

5. Discussion of Results

To find the variation of $\partial\alpha/\partial\sigma]_T$ over a wide range of temperatures requires an accurate determination of the nonlinear portion of the relation between e and T for constant σ. It is necessary to employ a nonlinear term in order to show any change in α with temperature. It became apparent early in the experimentation of finding the expansion curves that the data had insufficient accuracy to give a reliable value of the second order coefficient. The attention of the author was therefore focused on attempting to obtain an acceptable average value for $\partial\alpha/\partial\sigma]_T$ over the range of temperatures actually used.

The average value of $d(1/E)/dT$ was found to be 1.02 x 10^{-11} in.^{2}/lb-^{o}C and the average value of $\partial\alpha/\partial\sigma]_T$ was found to be 4.53 x 10^{-11} in.^{2}/lb-^{o}C, both over the range 78-450^{o}F.

Rosenfield and Averbach obtained values of $d(1/E)/dT$ = 1.2 and 0.8, $\partial\alpha/\partial\sigma]_T$ = 0.6 and 1.0 for two specimens respectively. The value of $d(1/E)/dT$ obtained in this work compares favorably with the results of Rosenfield and Averbach.

The value of $\partial\alpha/\partial\sigma]_T$ obtained in this work does not appear to be nearly as accurate as that of Rosenfield and Averbach. If one postulates that the value of $\partial\alpha/\partial\sigma]_T$ should coincide with that of $d(1/E)/dT$ within, say, one standard deviation of the latter as determined in this study, namely, in the range 1.02 $\pm$ 0.12 x 10^{-11} in.^{2}/lb-^{o}C, a probability analysis, given in Appendix I, shows that

there is a probability of 0.015 that $\partial\alpha/\partial\sigma]_T$, determined as $4.53 \pm 1.57 \times 10^{-11}$ in.^{2}/lb-^oC, would lie in this range in the absence of systemmatic error and presuming a Gaussian distribution of random error. The preceding strongly suggests the presence of systemmatic error. Attempts to discover such systemmatic error have been unsuccessful. However, as the next paragraph shows, there are sources of error sufficiently large that a small systemmatic bias could result in significantly altered results.

The cause of the large error in $\partial\alpha/\partial\sigma]_T$ in this work is attributed to the error involved in correction of the expansion of the extensometer with temperature. The nature of the gage thermal calibration was such that considerable uncertainty in the calibration factor was present due to furnace heat gradients, uncertainties of the stated properties of Vycor, and possible thermocouple error. The correction factor arising from this calibration was approximately ten times as great as the overall extensometer reading.

The use of H. Muir's [4] method of using resistance wire strain gages to determine reproducible expansion data with precision of $\pm$ 0.1 $\times 10^{-6}$ in./in.-^oC is apparently untried at elevated temperatures where the effects of gage creep and change of gage factor with temperature become significant.

Although the method used in this study was found to have many sources of error, it is believed that with improvement of furnace design, the use of the Tuckerman optical system would prove to be

equal to, or better than, the resistance wire strain gage system for determining the modulus of elasticity at temperatures to 500°F. Further attempts at using the Tuckerman optical system for thermal expansion tests however is not recommended.

The nature of the Tuckerman extensometer is such that fine accuracy is obtained at constant temperatures up to 500°F. In this study, the large temperature increments used resulted in a definite change in slope between the various stress-strain curves and therefore they aided in determining the value of $d(1/E)/dT$ over the whole temperature range.

In contrast to this, the large temperature range necessitated the use of low stress levels in order to avoid exceeding the elastic limit, and therefore hindered the establishment of differences in slope of the various strain versus temperature curves.

Rosenfield and Averbach, however, tested to high stress levels over a small temperature range, thereby increasing the accuracy of $\partial\alpha/\partial\sigma]_T$, while decreasing the accuracy of $d(1/E)/dT$. In their work they were not attempting to find the variation of these parameters with temperature, but only the value at one particular temperature. Their statement and that of Kunin and Kunin that the change in expansion coefficient is linear with stress over the range tested is necessarily an approximation appropriate within a small temperature range of testing.

In fact Sutherland, [10] in 1891, suggested that the ratio,

$Y = G/G_{AO}$ of the modulus of rigidity (G) to the modulus of rigidity at absolute zero (G_{AO}) versus the ratio, $X = T/T_M$ of absolute temperature T to melting temperature T_M for any material follows the line $Y = 1 - X^2$. Using the relation $G = E/2(1+\mu)$ valid for isotropic materials, we then find $G/G_{AO} = E(1+\mu_{AO})/E_{AO}(1+\mu)$. If the change of μ (Poisson's ratio) is assumed to be small, then E/E_{AO} should also follow the line $Y = 1 - X^2$. This would support the belief that $\left.\partial\alpha/\partial\sigma\right]_T$ will vary with temperature.

6. Variation and Error

Two major sources of error were present. The furnace design and construction was such that a heat gradient existed along the specimen. It is believed that although the mean value of the temperature indications (of the two thermocouples mounted on the specimen) was used, any deviation of this value from the average temperature existing between the extensometer knife edges would be sufficient to cause errors in measurements of the coefficient of expansion. If the average temperature was in error by $1^{\circ}F$, and assuming an α of 11×10^{-6} in./in.-°F, then over the 1.2-inch gage length of the extensometer with an extensometer constant of 3.3433×10^{-4} in./division an error would be present of $\left(1^{\circ}F \times 11 \times 10^{-6} \text{ in./in.}^{\circ}F \times 1.2 \text{ in.}\right) / \left(3.3433 \times 10^{-4} \text{ in./div}\right)$ $= 0.0395 \simeq 0.04$ divisions on the autocollimator scale (see Appendix II). Since the optical system allows readings to 0.02 divisions, this error would be large enough to influence strain readings.

The National Bureau of Standards has developed an air furnace with no heat gradients [11], but it has sealed and heated ends and no viewing port and therefore would not be suitable for measurements under tensile stress or for use of the optical system. A commercial type Marshall furnace with a viewing port and electrical taps from various parts of the heating coil to control current flow and therefore the temperature was available. This furnace however, proved to be unsuited for this study because the viewing port was only $\frac{1}{2}$ inch in diameter and a diameter of at least $1\frac{1}{2}$ inches was necessary to

accommodate the optical system. The furnace used in this study could be improved if a study were made of the heat loss through radiation and conduction at the pyrex viewing port and through the specimen pulling grips. The number of turns of Nichrome wire could then be adjusted in the region of the port to decrease the heat gradient. In addition, a more positive means of sealing the ends around the pulling grips could be devised to seal the furnace from air currents.

The second major cause of error is due to the expansion of the extensometer gage body itself. During thermal expansion tests the body of the gage will change with the temperature, thus causing the lozenge to rotate in opposition to the rotation caused by the expansion of the specimen material. To account for this, the extensometer was calibrated on a piece of Vycor as described in Part C of the preceding chapter. The correction factor arising from this calibration and variations in gage body thermocouple readings was sufficiently large to mask the expansion of the specimen and give unsatisfactory results for the thermal expansion curves. The calibration was subject to error due to heat gradients in the furnace as described above, as well as possibly in the stated value of α for Vycor. An analysis of the steps involved in determining $\partial\alpha/\partial\sigma\big]_T$ is shown below.

$e_g = C\Delta T$ = strain in gage body

$e_s = e_{s2} - e_{s1}$ = strain in specimen

$(e_s - e_g) = K\ DIV$ (DIV denotes divisions indicated on collimator reticle)

$(e_s - e_g)_1 + e_{g1} = e_{s1}$; $\quad (e_s - e_g)_2 + e_{g2} = e_{s2}$

$K\Delta DIV + C\Delta T_g = \Delta e_s$

$\alpha_a = \frac{\Delta e_s}{\Delta T_s} = \frac{K \Delta DIV_a + C \Delta T_{ga}}{\Delta T_{sa}}$ a indicates stress level a

ΔT_s = change in specimen temperature from initial temperature

ΔT_g = change in gage temperature from initial temperature

$\alpha_b = \frac{K \Delta DIV_b + C \Delta T_{gb}}{\Delta T_{sb}}$ b indicates stress level b

Let $T_{sa} = T_{sb}$

$$\partial\alpha/\partial\sigma = \frac{\alpha_b - \alpha_a}{\sigma_b - \sigma_a} = \frac{K(\Delta DIV_b - \Delta DIV_a) + C(\Delta T_{gb} - \Delta T_{ga})}{\Delta T_s \ \Delta\sigma}$$

$$= \frac{K}{\Delta T_s \Delta\sigma}(\Delta DIV_b - \Delta DIV_a) + \frac{C}{\Delta\sigma}\left(\frac{\Delta T_b - \Delta T_a}{\Delta T_s}\right)$$

It appears that the quantity$\left(\frac{\Delta T_{gb} - \Delta T_{ga}}{\Delta T_s}\right)$ would go to zero since $\frac{\Delta T_g}{\Delta T_s}$ would appear to approach unity, in which case $\partial\alpha/\partial\sigma = \frac{K}{\Delta T_s \Delta\sigma}(\Delta DIV_b - \Delta DIV_a)$ and the calibration factor C and gage temperatures would not enter into the final analysis. However, the ratio $\frac{\Delta T_g}{\Delta T_s}$ does not equal unity in the experimental data and although at first glance the variation would appear to be minor it does alter the solution considerably. For instance, if the values of ΔDIV, ΔT_g and ΔT_s from tests at 7000 psi and 12,000 psi are substituted into the relationship above,

$$\alpha_{12000} = \frac{K \times 1.14}{334.96} + \frac{C \times 339.55}{334.96}$$

$$\alpha_{7000} = \frac{K \times 1.22}{328.48} + \frac{C \times 315.00}{328.48}$$

$$\frac{\Delta\alpha}{\Delta\sigma} = \frac{\alpha_{12000} - \alpha_{7000}}{5000} = \frac{K(1.14 \times \frac{328.48}{334.96} - 1.22)}{328.48 \times 5000} + \frac{C(339.55 \times \frac{328.48}{334.96} - 315)}{328.48 \times 5000}$$

$$\frac{\Delta\alpha}{\Delta\sigma} = \frac{3.3433 \times 10^{-10}}{1.6424} \times .09 + \frac{.992 \times 10^{-10}}{1.6424} \times 2.014 = -1.832 \times 10^{-11} + 12.16 \times 10^{-11}$$

Suppose that there were a 2° error in the 315.00° temperature reading used in the preceding calculation. Then the difference of 20.14° appearing in the numerator in the second term would be reduced to 18.14° or increased to 22.14° and the term itself could show a variation from 10.96 to 13.37 x 10^{-11}, a range of 2.41 x 10^{-11}. A 2° error in temperature determination is certainly a possible error and this analysis shows that it can result in variations in calculated results that are themselves larger than the strain gage determinations. Thus it is readily apparent that small errors are able to alter considerably the results of $\partial\alpha/\partial\sigma]_T$ by changing the value of the second term above.

Since the correction involved was about ten times the value of the uncorrected extensometer reading, the author believes that the complete system of using the optical extensometer for thermal expansion tests is unsuitable. Notice however that these errors are predominant in thermal expansion measurements, but do not affect the tensile tests at constant temperature. Once the extensometer has steadied out at a particular temperature, its accuracy is maintained as previously discussed.

In addition to these major sources of error there are others. At elevated temperatures the heat gradient in the furnace sets up convection currents. These currents produce changes in air density with temperature and in the area of the observation window the light beam from the autocollimator is refracted. This refraction of the light

beam results in a blurring of the fiduciary spot, as viewed through the autocollimator. After considerable use of the instrument under these conditions, a facility was developed in obtaining readings that are considered repeatable with occasional errors of $\pm$ .04 divisions.

According to Paul H. Dike of the Leeds and Northrup Company [12], Chromel-Alumel thermocouples have limits of error as set by the wire manufacturer of $\pm$ 5F° in the range of 32 to 660°F. These limits however, are for random selection of stock wire. The wire used was duplex wire having a stated error $\pm$ 3F°.

In addition, there is a temperature error due to the degree of contact between the thermocouple and specimen, and due to radiation from or to the thermocouples.

7. Conclusions

These experiments tend to verify the value of $d(1/E)/dT$ as obtained by Rosenfield and Averbach. The value of $\partial\alpha/\partial\sigma]_T$ obtained in this study involves a large error and does not agree with the value of $d(1/E)/dT$ or with the result obtained by Rosenfield and Averbach.

The results obtained by use of the Tuckerman optical strain gage for tensile tests at elevated temperatures were considered good and the use of this procedure for determining the elastic modulus is worthy of continued investigation with improvement of furnace design.

Use of the Tuckerman Optical strain gage for thermal expansion measurements is not recommended due to the large change in extensometer gage length with changing temperatures. A continuation of this study could utilize the micrometer dilatometer method as defined by Lement, Roberts and Averbach [13].

The thermodynamic relationship stated in the Introduction is an interesting one which remains at this moment relatively unexplored. The work of Rosenfield and Averbach verified its validity only over a very limited temperature range and with a large factor of uncertainty. The present study, falling short of its objectives, merely tends to verify a part of Rosenfield and Averbach's data. Since the relationship is a fundamental one, its more general verification would add to understanding of the behavior of solid materials and the confidence with which one can postulate that (under short-time, low-load conditions) strain is a function only of temperature and stress.

Accordingly, it is recommended that continued work be done to verify this relation experimentally, perfecting the techniques which were found by the present study to be sound and devising alternate and appropriate techniques to replace those which this study finds to be unsound.

BIBLIOGRAPHY

1. Rosenfield, A. R. and Averbach, B. L., "Effect of Stress on the Expansion Coefficient," Journal of Applied Physics, Vol. 27, No. 2, February 1956.

2. Kunin, N. F. and Kunin, V. N., "The Influence of Stress on the Thermal Expansion of Deformed Metal," Fizika Metallov i Metallovedeniye, Vol. 5, No. 1, 1957.

3. Weaver, P. R., "An Optical Strain Gage for Use at Elevated Temperatures," Proceedings of the Society for Experimental Stress Analysis, Vol. IX, No. 1, 1951.

4. Muir, H., Sc.D Thesis, Department of Metallurgy, Massachusetts Institute of Technology (1953).

5. Carpenter, J. E. and Morris, L. D., "A Wire Resistance Strain Gage for the Measurement of Static Strains at Temperatures up to 1600°F," Proceedings of the Society for Experimental Stress Analysis, Vol. IX, No. 1, 1951.

6. Buckley, W. H. and Anderson, R. G., "Resistance Wire Strain Gages in Product Development," Proceedings of the Society of Experimental Stress Analysis, Vol. IX, No. 1, 1951.

7. Gorton, R. E., "Development and Use of High-Temperature Strain Gages," Proceedings of the Society of Experimental Stress Analysis, Vol. IX, No. 1, 1951.

8. Anderson, B. R., "Development and Application of High-Temperature Strain Gages for Measurements in Jet Engines," ASTM Special Technical Publication No. 230, Philadelphia, Pa., 1958.

9. Vycor Industrial Glassware by Corning, Bulletin No. B-91, Corning Glass Works, Corning, New York, 1960.

10. Sutherland, Philosophical Magazine, Vol. 32, No. 42, (1891).

11. Gray, Arthur W., "Production of Temperature Uniformity In An Electrical Furnace," Bulletin of the National Bureau of Standards, Vol. 10, No. 4, July 15, 1914.

12. Dike, Paul H., Thermoelectric Thermometry, Leeds and Northrup Company, Philadelphia, Pa., September 1954.

13. Lement, B. S., Roberts, C. S., and Averbach, B. L., "Determination of Small Thermal Expansion Coefficients by a Micrometric Dilatometer Method," The Review of Scientific Instruments, Vol. 22, No. 3, March 1951.

14. C.R.C. Standard Mathematical Tables, Twelth Edition, Chemical Rubber Publishing Company, Cleveland, Ohio, 1959.

15. Scarborough, J. B., Numerical Mathematical Analysis, The Johns Hopkins Press, Baltimore, Maryland, 1930.

APPENDIX I

NUMERICAL ANALYSIS OF DATA

A statistical analysis of curves of $e = e(T)$ (strain as a function of temperature), $e = e(\sigma)$ (strain as a function of stress), $d(1/E)/dT$, and $\partial\alpha/\partial\sigma]_T$ to determine the slopes of the various curves was accomplished using the Control Data Corporation 1604 digital computer, installed at the U. S. Naval Postgraduate School, using programs available in the computer program library.

The coefficients, a and b, in the equation $y = a + bx$ were determined by submitting the experimental test data to the statistical analysis generally referred to as "the method of least squares" a detailed study of which can be found in J. B. Scarborough's "Numerical Mathematical Analysis". [15]

The program for this reduction of data was titled "Line Fit" and was available in the computer program library. The program could not accommodate the weighting of points, but did give the standard deviation of the coefficients a and b with their mean values in the output.

The input was on punched cards. Each card will accommodate six experimentally determined points in the form $x_1y_1x_2y_2x_3y_3 \cdot \cdot \cdot \cdot \cdot x_6y_6$ where x and y are six digit numbers.

In the determination of the slopes of the tensile test curves approximately 25 points were used for the input data of each curve and the coefficients were determined for ten different curves at constant temperatures. In the determination of the slopes of the thermal expansion curves approximately 12 points were used for the

input data of each of six curves at constant stress. In the case of thermal expansion measurements, the raw data of divisions of strain versus temperature was first placed in the form of strain versus temperature by, 1) multiplying the divisions of strain by the appropriate constant (see Appendix II) to convert the value to strain (in./in.); 2) adding to this the product of ΔT of the gage body times the calibration factor (see Page 11 for discussion of the determination of calibration factor). Tensile test data was entered on the cards in the form of Type N indicator reading (μin./in.) and divisions of strain. The output value of the coefficient b was multiplied by an appropriate constant (see Appendix II) to give E.

The "Line Fit" program of least squares analysis computes the coefficients a and b by solving the following equations for the input data:

$$a = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$$

where n is the number of points and x and y are ordinate and absicca of the input data.

Probability analysis of error in $\partial\alpha/\partial\sigma]_T$.

In the discussion of results, it is stated that there is a 0.015 probability that the value of $\partial\alpha/\partial\sigma]_T$ using the experimental technique

of this work would fall within the limits of $d(1/E)/dT$. This value of 0.015 was determined by use of the normal curve of error and tables on page 244 of Reference [14]. To enter the tables, first find the number of standard deviations (t) required for the value of $\partial\alpha/\partial\sigma]_T$ to fall within the limits of $d(1/E)/dT$ as follows.

$$t = \frac{(4.53-1.14)}{1.57} = 2.16$$

where 4.53 is the mean value of $\partial\alpha/\partial\sigma]_T$, 1.14 is the mean value plus one standard deviation of $d(1/E)/dT$ and 1.57 is the standard deviation of $\partial\alpha/\partial\sigma]_T$, all in units of 10^{-11}/psi-°C. Entering the tables with t = 2.16, a value for the probability is obtained equal to 0.015.

APPENDIX II

DETERMINATION OF CONSTANTS

This Appendix includes the derivation of the various constants utilized in the reduction of data and referred to throughout the body of this work.

(1) Strain can be calculated by the equation:

$S = AxLxR/Ex1000$ (from Aminco-Tuckerman Optical Strain Gage System Instructions No. 750)

where (S) is strain in inches per inch, (A) is the autocollimator calibration factor (given by the manufacturer), (L) is lozenge size in inches, (R) is the difference in autocollimator readings, and (E) is the gage length.

$A = 1.003$

$L = 0.4$ inches

$E = 1.2$ inches

$$S = \frac{1.003 \times .4 \times R}{1.2 \times 1000} = 3.3433 \times 10^{-4}\ R \text{ in./in.}$$

(2) Stress may be calculated by the equation:

$\sigma = I/ZxA$ psi

where (I) is the SR-4 Type N indicator reading μin./in., (Z) is the SR-4 Type U load cell calibration factor μin./in.-lb, (A) is the cross-sectional area of the specimen in.^{2}.

$Z = 4.0$ in./in.-lb

$A = \pi D^2/4 = \pi \times .2501^2/4 = .04912$ in.^{2}

$\sigma = I/4.0 \times .04912 = I/.1965$ psi.